Graphing Calculators

in Calculus

(Using a **TI-89** Calculator)

Gettysburg College

Summary of Graphing Calculators in Calculus

You should be able to perform easily and efficiently all of the following tasks on your individual graphing calculator:

1. Do arithmetic calculations.
2. Define and evaluate functions.
3. Graph functions, and change the viewing window in meaningful ways.
4. Trace the graph of a function.
5. Find zeros of a function.
6. Find intersection points of the graphs of two functions.
7. Make a function table from a formula.
8. Find maxima and minima of a function.
9. Find the derivative of a function at a point, and graph derivative over an interval.
10. Find the definite integral of a function over an interval (LHS/RHS).
11. Graph the slope field of a differential equation, and sketch a solution curve.

#1: Using a TI-89 Graphing Calculator

Introduction

Graphing calculators and computer graphing software are indispensable tools in studying and doing mathematics. For this course you are **required** to have a graphing calculator available to you at all times during class, when doing your homework, and while taking exams. Although any calculator from the following list is acceptable, we **very highly recommend** that you use a calculator from the **Texas Instruments TI-83/84** series.

Texas Instruments TI-81, TI-82, **TI-83/84** series, TI-85, TI-86, or TI-89
Casio fx/cfx-7000/9000 series
Sharp EL-9000 series
Hewlett-Packard 48/49 series

Class demonstration, instruction, and discussion will all utilize a calculator from the **TI-83/84** series. Although the other listed calculators can perform most of the desired operations, those calculators might be more difficult to use on some operations that are expected in the course. Handouts are available for all of the **Texas Instruments** calculators that are listed. Although the TI-89 calculator has a Computer Algebra System and is much more powerful than any other TI-8x calculator, only the features in common with a TI-83/84 calculator will be discussed in the TI-89 handouts.

The purpose of handout #1 is to guide you through learning how to use the basic calculator features that are not related directly to functions and graphing. You will type or key-press the items in **bold**.

Keyboard Layout

Take a few moments to become familiar with the layout of the keyboard. The lower central portion has gray keys for the numerical digits. Arithmetic operation black keys are on the lower right side. The blue **arrow** keys form an oval on the top right. Just above most keys are two additional labels:

(1) A yellow label which you can access by first pressing and releasing the yellow **2nd** key on the top left of the keyboard, and
(2) A purple letter or a green label which you can access by first pressing and releasing the purple **alpha** key or the green **diamond** ◆ key (respectively) on the top left of the keyboard.

Note: If your screen is blank when you press the **ON** key in the lower left corner, hold down the green ◆ key, and then press and release the **plus** + key to increase the screen contrast until you see something on the screen. Use the green ◆ and **subtraction** − keys to reduce the contrast.

Approximate Mode

In order to use the TI-89 in a manner that is as close as possible to the TI-83, you need to set the calculator to work in the approximate mode, instead of the exact (symbolic) mode. To do this, press the black **MODE** key and then the blue **F2** key in the top row. Use the blue **arrow** keys to move the cursor down to the "*Exact/Approx*" line, and press the **right arrow** key. Use the **arrow** keys to select **APPROXIMATE**, and then press the **ENTER** key twice. **Leave your TI-89 in this mode for this course. All the handouts assume that your calculator is in the APPROXIMATE mode.**

Arithmetic Operations

Practice the following calculations along with other similar calculations of your own design until you are comfortable and proficient with basic arithmetic calculations. After you type each calculation to be done in the bottom *Entry Line*, press the **ENTER** key in the lower right corner of the keyboard. Use the **CLEAR**, **arrow**, and white-on-black **backspace** (delete to left) keys to make typing corrections. Enter each of the calculations as one formula without breaking the formula down into simpler pieces.

5 + 7	(Notice the answer "12" is on the right side of the screen, just above the *Entry Line*.)
3.6 − 8.25	(Use the subtraction key on the right side, just above the addition key.)
4 × 2	(Note that multiplication is displayed in the *Entry Line* by the " * " symbol.)
9 ÷ 4	(Note that division is displayed in the *Entry Line* by the " / " symbol.)
-4 + 9	(Use the negation **(-)** key on bottom row for negatives, NOT the subtraction key.)
3 × -6	(Use the negation key, NOT the subtraction key.)
2 + 3 × 4	(Where are the implied parentheses?)
(2 + 3) × 4	(Parenthesis keys are above the **7** and **8** keys.)
2 × 3 + 4	(Again, where are the implied parentheses?)
2 + 3 ÷ 4	(Implied parentheses?)
(2 + 3) ÷ 4	(Parentheses must be used to calculate $\frac{2+3}{4}$.)
2 ÷ 3 × 4	(What would you enter to calculate $\frac{2}{3 \times 4}$?)

When in doubt about which operations are performed first, either try a simple similar example or use parentheses to clarify what you intend. What should you enter to calculate $\frac{4+7}{2+3}$?

2 ^ 3	(The ^ key, just above the ÷ key, is for exponents: 2^3.)
(3 + 4) ^ 2	(Again, the parentheses are required.)
5 ^ -1	(Use negation key. Parentheses not required for the negative number exponent.)
$\frac{1}{2.4}$ $\frac{1}{2+3}$	(Try each twice: first use the ÷ key, and then use a power of -1.)
√ (4)	(Press yellow **2nd** key first, then **multiplication** key with yellow √ label.)
$\sqrt{12.25}$	(Parentheses required. Notice the √ gives an automatic opening parenthesis.)
$\sqrt{9 + 16}$	(Include the closing parenthesis.)
$\sqrt{9} + \sqrt{16}$	(Where are parentheses? Is this result the same as the last result?)
π ÷ 2	(To type π, press yellow **2nd** key first and then ^ key with yellow label "π".)
2 × π	
2 π	(Omit the × key on this and the next three examples;
3 √ (4)	these four examples illustrate "implied" multiplication.)
2 (3 + 4)	
1 / 2 π	(Is the result what you expected?)

Last Entry

Type again the calculation **2 (3 + 4)** and press the **ENTER** key. Notice that your last formula entered stays in the *Entry Line* for you to make changes. Use the **arrow** and ⇦ (**backspace**) keys to change the "3" to a "5" in order to compute **2 (5 + 4)**. Press the **ENTER** key as usual to carry out the calculation. If you would like to edit a formula that was several entries back, press the blue **up arrow** key repeatedly to highlight that formula in the *History Area*, and then press the **ENTER** key.

While you are typing or editing a formula, if you press the yellow **2nd** key and then the ⇦ key with the yellow label **INS**, then the calculator switches between the “insert” and “typeover” modes. The cursor in “typeover” mode is a flashing rectangle over the character to be typed over, and the cursor in the “insert” mode is a flashing vertical line at the point of insertion.

Note: Watch what happens to the *Status Line* at the bottom of the screen when you press the yellow **2nd** key. Press the **2nd** key again and watch. If you ever mistakenly press **2nd**, you can cancel by pressing **2nd** again. Look at the *Status Line* to see whether **2nd** is activated. Similarly for the ◆ key.

Variables

Type the calculation: **2 + 3 ENTER**. Notice that the answer is 5. Now press the × key and the **4** key. The *Entry Line* reads "ans(1)*4". What is the result when you press **ENTER**?

The "ans(1)" is a variable that stores your last calculated value (as opposed to your last entered formula). Now type **50 – ans(1)** (press **2nd** key and then **ANS** key in lower right corner of keyboard), and then press **ENTER**. The value of the variable "ans(1)" changes after each new formula calculation.

If you wish to calculate again the last formula entry without editing it, just press the **ENTER** key -- as many times as you wish to perform the calculation.

Example: Press **3** and then the **ENTER** key. The value of ans(1) is now 3.
Type **2 × ans(1) ENTER**. (Use the **2nd** and **ANS** keys.) The value of ans(1) is now 6.
Press the **ENTER** key repeatedly to see the value of ans(1) repeatedly doubled.

The result of each calculation is automatically stored in the variable ans(1). Values can also be stored in single-letter variables by using the **STO▷** key located in the lower left corner of the keyboard. Carry out the following example entries and observe what happens:

5 STO▷ a	**ENTER**	(Press the **alpha** key and then the = key with purple label **A**.)
2 + a	**ENTER**	(The value stored in variable "a" is used in the calculation.)
a ÷ 2 STO▷ b	**ENTER**	(The value of a/2 is displayed and also stored in variable b.)
a × b	**ENTER**	(Current values of variables a and b are used in calculation.)
7 STO▷ a	**ENTER**	(The value of a is changed, but formula a/2 is not recalculated.)
b	**ENTER**	(Value of b did not change when a new value was stored in a.)

Experiment with other examples using variables to be sure you understand how they work in formulas and calculations. Variable names are sensitive to using upper and lower case letters. To type upper case letters, press the **shift** key (the white-on-black upward arrow) before the **alpha** key. As you experiment with the **shift** and **alpha** keys, watch the *Status Line* at the bottom of the screen.

You should practice repeatedly all the features discussed in this handout (and each later handout) until you are comfortable and proficient with them. You must be able to use these features easily and efficiently. These handouts show you the most important features you will need in calculus. Consult your calculator manual for further details and features.

#2: Functions and Graphing on a TI-89 Calculator

The purpose of handout #2 is to learn how to define, evaluate, and graph functions with your calculator. A TI-89 calculator can work simultaneously with up to 99 user-defined functions. Our first example will be the linear function $y = f(x) = x - 2$. Type or key-press the items in **bold**.

DEFINE the function by pressing the ◆ and **Y=** keys (green label on the **F1** key) in the upper left corner, moving the cursor to the "y1=" line, typing **x − 2** in the *Entry Line*, and pressing **ENTER**. Use the separate **X** key (on the left side, below the **HOME** key) to type the independent variable. (Remember you can use the **CLEAR**, ⇦, **INS**, and **arrow** keys to edit.) Press the **HOME** key to get back to the *Home screen*.

Note 1: In function mode you must use the letter "*x*" as the name of the independent variable for any "*y* =" function, no matter what the independent variable might be named in your actual problem. For example, if your problem uses $g(p) = p^2$, you must use "y1(x) = x^2" on the calculator.

EVALUATE the function you have defined by using function notation with "*y1*" as the name of the function entered in the *Entry Line* of the *Home screen*. To type the letter "y", use the separate **Y** key to the right of the **X** key. Practice the following examples, and observe the output.

y 1 (3)	(This is standard function notation using "*y1*" instead of "*f*".)
y 1 (6)	(Remember you can simply edit your *Last Entry* in the *Entry Line*.
5 y 1 (4) + 3	(Notice the implied multiplication.)

GRAPH the function you have defined by selecting **WINDOW** (the ◆ and **F2** keys) and **ZOOM** (the **F2** key again), and then choosing option **4** for **ZoomDec**. The graph of your function is plotted in a viewing window that extends from -7.9 to 7.9 along the *x*-axis and from -3.8 to 3.8 along the *y*-axis. Tick marks are placed every 1 unit along each axis. Press the **WINDOW** key again to see these specifications for the viewing window. The variables *xmin*, *xmax* and *ymin*, *ymax* describe the extent of each coordinate axis. The variables *xscl* and *yscl* describe the numerical distance between tick marks along these axes. The values of the window variables may be changed, but we will not do so at this time.

Note 2: A menu option may be selected either by pressing the option number or by highlighting the option using the **arrow** keys and then pressing the **ENTER** key.

Press the **GRAPH** key (◆ and **F3** keys) to see the graph again. Use the **arrow** keys to move the *free-moving cursor* (+ sign with circle) around the viewing window. Notice the x- and y-coordinates of the point at the center of the cursor. Use the free-moving cursor to write down the coordinates (with comma and parentheses) of three sample points of your choice from the graph of the function.

Now select **TRACE** (the **F3** key). You will see the *trace cursor* (flashing) on the graph of the function. Notice also the x- and y-coordinates of the graph point located at the cursor. Use the **right** and **left arrow** keys to move the trace cursor along the graph of the function. Notice in this example that the y-coordinate is always 2 less than the x-coordinate. If you press the **ENTER** key, the viewing window will be made to center on the trace cursor. Press the **WINDOW** key to see the new values of the viewing-window variables. Select **ZOOM** and **ZoomDec** again to reset the original viewing window. Experiment with what happens when you press the **ESC**, **QUIT** (the **2nd** and **ESC** keys), **GRAPH**, **TRACE**, **CLEAR**, and **ZOOM** **ZoomDec** keys from various different screens.

Note 3: The trace arrow keys restrict the possible x-coordinates of the points that can be specifically computed since the trace cursor moves in jumps from one screen pixel (tiny, square picture element) to another. To compute the value of the function and plot the corresponding graph point at arbitrary values of x, first **GRAPH** the function and then:

Press **MATH** (the **F5** key) and then the **1** or **ENTER** key to select "Value".

Type the desired value of x (**1.325** for example) and press the **ENTER** key.

(*Alternately*, just press the **TRACE** key, type **1.325**, and press the **ENTER** key.)

Note 4: When tracing a function, each press of a left or right arrow key moves the trace cursor a certain number of pixels left or right, as specified by the value (1-10) of the variable *xres* defined in the **WINDOW** screen. These are the only values of x at which the function is actually computed for making the graph. Smaller values of *xres* will give better graphs, but will take longer to graph.

Note 5: More than one function can be defined and graphed at the same time. We will use $y = g(x) = 1 - x$ as our second example function.

Press **Y=** and move the cursor to the "*y2*=" line. Enter the function **1 − x**, and select **GRAPH**.

Press **TRACE**. Move trace cursor to the point where x=.7, and watch what happens when you press **up** and **down arrow** keys. Notice the expression in upper right corner of screen.

Now choose **MATH** **Value** and enter **2.637** for the value of x. Again watch what happens when you press the **up** and **down arrow** keys.

#3: Changing the Viewing Window on a TI-89 Calculator

The purpose of handout #3 is to learn how to move the viewing window around the coordinate plane. Our two examples are the following exponential functions: $y = f(x) = 0.5\,(2^x)$ $y = g(x) = 2\,(0.4)^x$

SELECT functions for graphing by pressing the **Y=** keys, **CLEAR**ing any previously defined functions, and entering **.5 × 2 ^ x** for y1 and **2 × .4 ^ x** for y2. Since the y1 and y2 functions are checked, they are selected for graphing. Choose **ZOOM** (**F2** key) and **ZoomDec**, and notice the "BUSY" indicator in the lower right corner of the screen indicating that the calculator is busy working.

Go back to **Y=**. Move the cursor to the "y1=" line, and press the **F4** key to uncheck the *y1* function. Press **GRAPH** and observe only one function graph (which one?). Although the unchecked *y1* function is not plotted, it is still defined and can be used and evaluated from the *Home screen*.

Press **Y=**, use the **F4** key to make a check on the line "y1=", and press **GRAPH**. While graphing, press the **ENTER** key to pause and resume (watch the *Status Line*), or the **ON** key to stop. Observe the possible values of *x* and *y* when you use the **arrow** keys to move the free-moving cursor.

CHANGE the viewing **WINDOW** to make *x* vary from -3 (negation key, not subtraction key) to 4 with tick marks every 1 unit and to make *y* vary from -2 to 12 with tick marks every 2 units. **GRAPH**. What is different about the values of *x* and *y* when you now use the **arrow** keys to move the free-moving cursor? The previous **ZOOM ZoomDec** had set the window variables so that the pixel steps are 0.1 in all directions. Most other choices for values of the window variables lead to more awkward sizes for the pixel steps. **TRACE** uses the pixel steps for *x*-coordinates and computes the function values for *y*-coordinates. On the graphing screen, **MATH Value** allows you to type in whatever *x*-coordinate you wish and computes the function value to give the *y*-coordinate. Experiment with trying different values of the window variables and using **TRACE** and **MATH Value**.

ZOOMING is a shortcut to making certain kinds of changes in the values of the window variables. Choose **ZOOM ZoomDec** to set the particularly nice values for the window variables. We can also zoom in and out from whatever is our current viewing window. To set zooming factors press the **ZOOM** key, and then press the **up arrow** key and **ENTER** to choose **SetFactors**. Change both the *xFact* and *yFact* variables to have the value **2** instead of 4 (use **arrow** keys between variables).

ZOOM IN: Press the **ZOOM** key (**F2**), press the **2** key to choose **ZoomIn**, press the **ENTER** key to accept the origin as the new zoom center. Use the **arrow** keys to move the free-moving cursor, and observe that the pixel steps of 0.05 are exactly half of the previous 0.1 pixel steps. The numerical distances between tick marks were also changed by the same *Zoom Factors*. Press **WINDOW** and notice that zooming has multiplied the previous min, max, and scl values by ½. Go back to view the **GRAPH**.

ZOOM OUT: Press **ZOOM** and choose **ZoomOut**. Use the **arrow** keys to move the move the free-moving cursor to the point $(1.5, 1)$. Press the **ENTER** key to zoom out centered on this point. Observe how the axes are off-center. When the graphing is finished, press **ZOOM**, **ZoomOut**, and **ENTER** to zoom out again on the same point. Finally, press **CLEAR** to erase the coordinates. Press **WINDOW**, note the new scale values, and go back to **GRAPH**.

ZOOM BOX: An alternate method of zooming in is to form a rectangular box to be the new viewing window. Let's apply this method to zoom in on the point of intersection of our two example function graphs. Press **ZOOM** and then **ENTER** to choose **ZoomBox**. Use the **arrow** keys to move the free-moving cursor to any corner of the new viewing rectangle desired. Press the **ENTER** key. Then repeat the **ZOOM** and **ZoomBox**, use the **arrow** keys again to move the cursor to the opposite corner of that rectangle, and press **ENTER**. Keep making new zoom rectangle selections repeatedly until the x- and y-coordinates of the cursor do not change in, say, the fifth decimal place when the cursor is moved one pixel step in each direction. Write down (using comma and parentheses) the intersection point with coordinates rounded to four decimal places. *Answer*: (0.8614, 0.9084)

Note 1: After you have zoomed in and/or out to obtain the viewing window you desire, the resulting scale values may not look very nice. Ffor example, they may be messy decimal values. Also, the ZoomBox command does not change the scale values, so you may not even see tick marks. To correct this, just press **WINDOW** and enter more suitable values for *xscl* and *yscl*. You may also wish to "clean up" the values of *xmin*, *xmax*, *ymin*, and *ymax*. Press **GRAPH** to view the results.

Note 2: If you wish grid dots displayed within the viewing window, select **WINDOW**, **Tools** (**F1** key), and **Format** (choice # **9**). Use **arrow** keys to go to the “Grid” line, right, and down to select **ON**; then press **ENTER** twice. Finally, press **GRAPH** to see the result. The same steps can also be used to turn off the grid dots and make other changes in how the viewing window looks.

#4: Formula Tables and Data Plots on a TI-89 Calculator

The purpose of handout #4 is for you to learn how to use formula tables, data tables, and data plots. Our two example exponential functions have formulas $y = 0.5\,(2^x)$ and $y = 2\,(0.4)^x$. You should press the **Y=** key and enter these formulas for y1 and y2 as before. The example data table is given to the right. We will enter this data into the calculator after we learn how to use a formula table.

x	y
-2	3.05
-1	2.55
0	1.94
1	1.64
2	1.25
3	1.07
4	0.84

FORMULA TABLE setup is started by pressing the **TblSet** key (green ◆ and **F4** keys). Enter **0** for the value of *tblStart* to begin the values of the independent variable x in the table and **1** for the value of Δ*tbl*, to give the step size for x. (Leave *Graph* <-> *Table* as "Off" and *Independent* as "Auto".) Press **ENTER** to save. Now press the **TABLE** key (green ◆ and **F5** keys) to view a table of values for the functions selected and defined by the given formulas. Use the **arrow** keys in a natural way to scroll through the table. Notice that a more precise value of the highlighted entry appears at the bottom of the screen. Press **TblSet** as before. Change the starting value of x to -2 and the step size to 0.1. Press **ENTER** to save and **TABLE** to view the table. Use all the **arrow** keys to explore the new table.

DATA TABLE entry is started by pressing the **APPS** key and selecting **Data/Matrix Editor** and **New**. Leave *Type* as "Data" and *Folder* as "Main". Use the **alpha** key to type **d1** for the example variable name, and press **ENTER** twice. (Next time you edit data, select **Current**.) Each column in the table is a list of data values. Enter the "x" data values from the example data table above into the "c1" column. Press the **ENTER** or **down arrow** key to go to the next value. Press the **right** and **up arrow** keys to go to the top of the "c2" column, and enter the "y" data values from the example data table above. Use **arrow** keys to move to any entry you wish to correct. While you are editing an individual entry, use the **CLEAR**, ⇦, **DEL**, and **INS** keys. If an entry is highlighted and you have not started to edit the entry, the ⇦ key removes that entry from the list and pushes others below it up. Pressing the **F6** (**2nd F1**), **right arrow**, and **cell** keys will insert an undefined entry (which can be edited) and push other entries down the column. Experiment with the editing features until you are comfortable with how they work. It is important to remember that all of the numbers in a data table are there by individual entry, not from a formula calculation.

PLOT THE DATA from the data table as follows:

1. Uncheck all of the functions defined by formulas under **Y=**. You may do this either separately for each function by pressing the **F4** key while highlighting the function or all at once by pressing the **F5** key and choosing **All Off** or **Functions Off**.

2. Go back to the data editor (**APPS**, **Data/Matrix Editor**, and **Current**). Press the **PlotSetup** key (**F2**), highlight "Plot1", and press the **Define** key (**F1**). Use **arrow** and **ENTER** keys to select the following options

Plot Type:	**Scatter**	
Mark:	**Box**	
x:	**c1**	(*Caution*: Sometimes there is an automatic **ALPHA** key pressed.)
y:	**c2**	

   Press the **ENTER** until you get back to the data table.

3. Select **WINDOW**, **ZOOM**, and **ZoomData** to automatically set a viewing window that includes all of the data points. Press **WINDOW**, enter suitable scale values, and modify the other window variables as desired. For example, change variables *ymin* to **0** and *ymax* to **3.5**. Press **GRAPH** to view the data in your new window. Notice that the graph of the data looks somewhat like a decreasing exponential function.

MODEL THE DATA using a function defined by a formula. You can use trial and error to determine possible values for the parameters *a* and *b* in the family of exponential functions with formula $y = b \cdot a^x$ so that the graph of the function fits the data points as best as you can make it. The specific formula should be entered as one of the selected **Y=** functions. For example, start by entering the formula **2.1 × 0.7 ^ x** for the function y3. View the **GRAPH** to see how well this specific model fits the given data. Modify the values of the parameters *a* and *b* to try to get a better fit.

TURN OFF THE DATA PLOTS by pressing the **Y=** key and choosing **Data Plots Off**.

IMPORTANT: BE SURE YOU TURN OFF THE DATA PLOTS WHEN YOU FINISH THIS SECTION.

Note: A family of functions can be graphed using a *list* for the parameter. For example, graph the family $f(x) = x^p$ by entering y4= **x ^ { .5, 1, 1.5, 2 }** ({ } are yellow labels on parenthesis keys.)

#5: Left and Right Sums on a TI-89 Calculator

The purpose of handout #5 is to implement the computation of left- and right-hand sums on the calculator. Suppose we wish to estimate the area under the graph of $f(x)$ over the interval from $x=a$ to $x=b$ by forming the appropriate left-hand and right-hand sums for various values of n. For any choice of n, the increment in x would be $h = \frac{b-a}{n}$, and the equally spaced values of x would be

$$\begin{cases} a = x_0 & \\ \quad x_1 = x_0 + h = a + h \\ \quad x_2 = x_1 + h = a + 2h \\ \quad x_3 = x_2 + h = a + 3h \\ \quad \vdots \qquad \vdots \qquad \vdots \\ b = x_n = x_{n-1} + h = a + nh \end{cases}$$

Given $f(x)$, a, b, and a choice for n, the left- and right-hand sums, L and R, may be written:

$$L = \sum_{i=0}^{n-1} f(a+ih)\cdot h \quad \text{and} \quad R = \sum_{i=1}^{n} f(a+ih)\cdot h, \quad \text{where } h = \frac{b-a}{n}.$$

Notice that the only dissimilarity between the left-hand sum and the right-hand sum is in the lower and upper limits of summation. Finally, as the value of n becomes arbitrarily large, the values of L and R both approach the area under the graph of the function $f(x)$ over the interval $a \le x \le b$

To implement the computation of left-hand sums on the calculator, we will write a program, that is, a list of the steps for the calculator to do. We need to type the program into the calculator only once. Then to compute a left-hand sum, we define the particular function and request the calculator to perform the program steps ("execute the program"). We can similarly implement the computation of right-hand sums. A brief *outline* of the program we will create later is as follows:

a. Prompt for values of the parameters a, b, and n.
b. Compute the increment, $h = \frac{b-a}{n}$.
c. Starting with a sum of zero, repeatedly add each new product $f(a+ih)\cdot h$ to the previously accumulated sum.
d. Display the final sum.

<u>CREATE NEW PROGRAMS</u> by typing program instructions into the calculator:

1. Press the **APPS** key, and select **Program Editor** and **New** to create a new program.
2. Type in the name of your program under *Variable:* **lhs**, and press the **ENTER** key twice. (Note that the cursor is set into the **ALPHA** mode as you type the variable name, so you can just press the keys corresponding to the letters in the program name without using the **ALPHA** key.)

3. Type in the program itself using the key-press hints given in parentheses. Press the **ENTER** key at the end of each line. Remember to press the **alpha** key to type an alphabetic letter and the **shift** and **alpha** keys to type a capital letter (**alpha** key also turns off alphabetic mode). If you need to make corrections, use **CLEAR**, ⇦, **INS**, **DEL**, and **arrow** keys in the natural way.

lhs( )	(**Arrow down** to the third line**:** colon with blank line.)
Prgm	(The comma (**,**) key is just above the **9** key.)
Prompt a , b , n	(Press the **I/O** key (**F3**), and select **Prompt**.)
(b – a) / n → h	(Press the **STO▸** key to type "→".)
0 → L	(**L** accumulates sum of products, initially 0.) (**alpha** at end)
For i , 0 , n – 1	(Press the **Control** key (**F2**) and select **For**... **EndFor**.)
L + y1 (a + i * h) * h → L	(Use **shift** and **alpha** to type **L**, and then press **alpha**.)
EndFor	(**Arrow down** and press the **ENTER** key to bypass this line.)
Disp L	(Press the **I/O** key (**F3**), and select **Disp**.) (**alpha** after **L**)
EndPrgm	(This line was type automatically for a new program.)

4. When you have finished typing in the program, press the **2nd QUIT** key.
5. To type in the program for computing right-hand sums, repeat the instructions in steps 1-4. Use **rhs** for the name, replace "**L**" by "**R**" everywhere, and change the For statement to: **For i , 1 , n**
6. If you need to modify a program after you have typed it into the calculator, press the **APPS** key, and select **Program Editor** and **Open**. Use the **arrow** and **ENTER** keys to select the program you want to modify. Use the **INS**, **DEL**, and **arrow** keys in the natural way to help you make any changes. When you are finished with modifications, press the **2nd QUIT** key.

<u>EXECUTE PROGRAMS</u> lhs and rhs to approximate, for example, the area under the graph of $f(x) = x^3$ over the interval from $x = 1$ to $x = 3$, using $n = 100$ subdivisions:

1. Enter the formula x^3 as the y1 function. Remember that lhs and rhs were written to use only the y1 function.
2. Type **lhs( )** in the entry line of the Home Screen, and press **ENTER** key to execute the program.
3. Enter the values **1**, **3**, and **100** (one value at each question mark) for the values of the parameters a, b, and n.
4. <u>Wait</u> until the calculation is done: the "BUSY" indicator stops. (Answer should be 19.7408.)
5. Press **ESC**, and repeat steps 2-4 to compute the corresponding rhs. (Answer should be 20.2608.)
6. Press the **ESC** key, and enter the formula **(L + R) / 2** on the Home Screen to compute the average of the left- and right-hand sums. (Answer should be 20.0008.)

#6: Calculus Features on a TI-89 Calculator

In addition to the operations that we have previously learned (see earlier handouts), there are some powerful features on a TI-89 calculator that provide shortcuts to various calculus calculations. There are two major ways to carry out most of these calculations:

a. Commands executed from the Home Screen or included in **Y=** function definitions, and
b. Interactive calculations on the Graph Screen.

Consult your TI-89 calculator manual for further details of the features discussed below.

For this handout, define example functions $f(x) = x^3 - 3x^2 + 2.5$ and $g(x) = x - 0.4$ respectively. **GRAPH** both functions, and select **ZOOM** and **ZoomDec**. Use ▹ and ◃ to change the value of x. Type or key-press items in **bold**.

1. *Evaluate a function*. For example, compute $f(1.6183)$ and $g(1.6183)$. (Ans: -1.11853 and 1.2183)

   Home Screen: Enter **y1(1.6183)** and **y2(1.6183)** as we did in an earlier handout.

   Interactive: **GRAPH**, **Math** (**F5**), and **Value**, xc: **1.6183** Use ▵ and ▿ to switch functions.

   Notes: **GRAPH** and **Trace** (**F3**) gives function evaluation at pixel & entered values of x.
   TABLE provides another way to evaluate functions.

2. *Find zeros of a function*. For example, the first of three zeros of $f(x)$ lies between -1 and 0 at about -0.5 for x.

   Home Screen: Use **F2**: **nSolve(y1(x) = 0, x = -.5)** See x = -0.810038 is a zero of $f(x)$.
   Edit the "nSolve" command to find the other two zeros.

   Interactive: **GRAPH**, **Math** (**F5**), and **Zero**. Use ▵ and ▿ to select y1 (upper right corner).
   Lower Bound? xc: **-1** Upper Bound? xc: **0** (other values possible)
   See value x = -0.81038 as a zero of $f(x)$.

3. *Find intersection points of two graphs*. For example, the first of the three intersection points of $f(x)$ and $g(x)$ is at about -1 for x.

   Home Screen: Use **F2**: **nSolve(y1(x) = y2(x), x = -1)**
   See x = -0.987381 for the intersection point. **QUIT** to the Home Screen.
   y1 (ans(1)) **ENTER**. See y = -1.38738 at the intersection point.

   Interactive: **GRAPH**, **Math** (**F5**), and **Intersection**
   Use **arrows** and **ENTER** to select 1st and 2nd curves.
   Lower Bound? xc: **-1.5** Upper Bound? xc: **-.5** (other values possible)

   Note: Repeated use of **TRACE** and **GRAPH ZOOM** (as we did in earlier handout) is a much longer process for finding zeros of a function and intersection points.

4. *Find local maxima and minima of a function on an interval.* For example, find the <u>single</u> local maximum or <u>single</u> local minimum of the function $f(x)$ on the interval [1,3], if there is one.

Home Screen: **fMax(y1(x), x) | x ≥ 1 and x ≤ 3** (Use **F3**, and **2nd MATH** (**5** key) **Test**.)
The graph shows the "x=1" answer is incorrect. Try again with another interval.
fMax(y1(x), x) | x ≥ 1.5 and x ≤ 3 (The "x=3" answer is more reasonable.)
fMin(y1(x), x) | x ≥ 1 and x ≤ 3 (Substitute answer value "2" into $f(x)$.)
y1(2) (See -1.5 as that local minimum value of $f(x)$.)

Interactive: **GRAPH**, **Math** (**F5**), and **Minimum** Select function y1, upper right corner.
Lower Bound? x= **1** Upper Bound? x= **3** (other values possible)
See x = 2 and y = -1.5 as the local minimum point for $f(x)$ over the interval [1,3].

5. *Find and graph the derivative of a function.* For example, compute $f'(2.5)$ and graph $f'(x)$.

Home Screen: **nDeriv(y1, x) | x=2.5** Use **Calc** (**F3** key) and **arrow** down to **nDeriv(**
See 3.75 as approximate value of $f'(2.5)$.

Note: nDeriv uses a central difference quotient with 0.001 as the value of h.
nDeriv(y1, x, .0005) | x= 2.5 (Uses 0.0005 as the value of h.)

Y= Screen: Define y3= **nDeriv(y1(x), x)** (Use **2nd MATH** and **Calculus**.)
Zoom and **ZoomDec** (To graph y1, y2, y3.)
y3 can be used as $f'(x)$ for computation and graphing. Now <u>deselect</u> y3 in **Y=**.

Interactive: Select **GRAPH**, **Math** (**F5** key), **Derivatives**, and **dy/dx**
Use △ and ▽ to select function y1 (see upper right corner).
xc: **2.5** and **ENTER**. See "3.75" as approximate value of dy/dx at given point.

6. *Find the definite integral of a function over an interval.* For example, compute $\int_{0.2}^{2.0} f(x)\,dx$.

Home Screen: **nInt(y1(x), x, .2, 2)** and see 0.5076 as value of definite integral. (Use **Calc**.)

Interactive: **GRAPH**, **MATH**, and $\int f(x)\,dx$ Select function y1 (see upper right corner).
Lower Limit? xc: **.2** Upper Limit? xc: **2** (**ENTER** to accept each limit)
See 0.5076 as value of integral. Note the corresponding signed area is shaded.

7. *Draw on the graphing screen.*

Use **Draw** (**2nd F6**) and **ClrDraw** to clear any previous drawings. <u>Deselect</u> y2 in **Y=**.
GRAPH Draw DrawFunc y1(x) − 2 ENTER will provide a temporary graph of $f(x) - 2$.
Draw ClrDraw when done.
Draw DrawInv y1(x) ENTER will make temporary graph of inverse relation for $f(x)$ .This example inverse relation is not a function since $f(x)$ is not invertible. (**ClrDraw** when done.)

8. *Draw tangent lines to graph of a function.* For example, draw the tangent line to the graph of $f(x)$ at x = 2.2 . Be sure that only $f(x)$ is selected on the **Y=** screen.

Graph Screen: **GRAPH Math Tangent** Select function y1. Set xc: **2.2** and **ENTER**.
Press **Regraph** (**F4** key) to clear the tangent line.

#7: Differential Equations on a TI-89 Calculator

The purpose of handout #7 is to implement graphing the slope field of a differential equation and sketching an approximate solution curve using Euler's method. Type or key-press items in **bold**.

1. In the window $-4 \le x \le 4$ and $-3 \le x \le 3$, let's use the example differential equation $\frac{dy}{dx} = x + y$

2. Press the **MODE** key, set the *Graph* to **DIFF EQUATIONS**, and press **ENTER** twice.

3. The TI-89 uses "*t*" instead of "*x*" for the independent variable in differential equations. Press ◆ **Y=** and enter the expression **t + y1** for the y1′=. Note the *x* and *t* variables have separate keys that don't require the alpha key. By way of example, enter t0= **-1** and yi1= **.2**.

4. From within the ◆ **Y=** screen, press the ◆ | key, set the *Solution Method* to **EULER**, set the *Fields* to **SLPFLD**, and press the **ENTER** key to accept the Mode changes.

5. Press ◆ **WINDOW** and enter the appropriate values for the *x* and *y* intervals, using a scale value of 1 in each direction. Other parameters should be set as follows:

t0= **-1**	tmax= **10**	tstep= **.1**	tplot= **-1**
ncurves= **0** diftol= **.001**		fldres= **20**	

6. Press ◆ **GRAPH**, and watch the slope field and solution curve being graphed.

7. Press ◆ **Y=**, edit the coordinate values of the initial point (t0, yi1) to be **-1** and **.4**, and press ◆ **GRAPH**.

8. Again press ◆ **Y=** and edit the coordinate values of the initial point. To show multiple solution curves, type in a sequence of values for the yi1=, such as **{ .2 , .4 }**, and press ◆ **GRAPH**.

9. To plot only solution curves without the slopefield: from within the ◆ **Y=** screen or the ◆ **GRAPH** screen, press the ◆ | key and change the *Fields* to **FLDOFF**.

10. When you are finished with differential equations problems, be sure to put your calculator back to its usual mode. Press the **MODE** key, set the *Graph* to **FUNCTION**, and press **ENTER** twice.