Graphing Calculators

in Calculus

(Using a **TI-86** Calculator)

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Summary of Graphing Calculators in Calculus

You should be able to perform easily and efficiently all of the following tasks on your individual graphing calculator:

1. Do arithmetic calculations.
2. Define and evaluate functions.
3. Graph functions, and change the viewing window in meaningful ways.
4. Trace the graph of a function.
5. Find zeros of a function.
6. Find intersection points of the graphs of two functions.
7. Make a function table from a formula.
8. Find maxima and minima of a function.
9. Find the derivative of a function at a point, and graph derivative over an interval.
10. Find the definite integral of a function over an interval (LHS/RHS).
11. Graph the slope field of a differential equation, and sketch a solution curve.

#1: Using a TI-86 Graphing Calculator

Introduction

Graphing calculators and computer graphing software are indispensable tools in studying and doing mathematics. For this course you are **required** to have a graphing calculator available to you at all times during class, when doing your homework, and while taking exams. Although any calculator from the following list is acceptable, we **very highly recommend** that you use a calculator from the **Texas Instruments TI-83/84** series.

Texas Instruments TI-81, TI-82, **TI-83/84** series, TI-85, TI-86, or TI-89
Casio fx/cfx-7000/9000 series
Sharp EL-9000 series
Hewlett-Packard 48/49 series

Class demonstration, instruction, and discussion will all utilize a calculator from the **TI-83/84** series. Although the other listed calculators can perform most of the desired operations, those calculators might be more difficult to use on some operations that are expected in the course. Handouts are available for all of the **Texas Instruments** calculators that are listed.

The purpose of handout #1 is to guide you through learning how to use the basic calculator features that are not related directly to functions and graphing. You will type or key-press the items in **bold**.

Keyboard Layout

Take a few moments to become familiar with the layout of the keyboard. The lower central portion has the numerical digits. Arithmetic operation keys are on the lower right side. The **arrow** keys form an oval on the top right. The **F1-F5** and **M1-M5** keys show as menu items on the screen. Just above most keys are printed two additional labels:

(1) A yellow label which you can access by first pressing and releasing the yellow **2nd** key on the top left of the keyboard, and
(2) A light blue letter or symbol label which you can access by first pressing and releasing the light blue **ALPHA** key on the top left of the keyboard.

Note: If your screen is blank when you press the **ON** key in the lower left corner, press and release the yellow **2nd** key once and then hold down the **up arrow** key to increase the screen contrast until you see something on the screen. Use the **2nd** and **down arrow** keys to reduce the contrast.

Arithmetic Operations

Practice the following calculations along with other similar calculations of your own design until you are comfortable and proficient with basic arithmetic calculations. After you type in each calculation to be done, press the **ENTER** key in the lower right corner of the keyboard. Use the **CLEAR**, **arrow**, and **DEL** (delete) keys to make typing corrections. Enter each of the calculations as one formula without breaking the formula down into simpler pieces.

5 + 7	
3.6 − 8.25	(Use the subtraction key on the right side, just above the addition key.)
4 × 2	(Note that multiplication is displayed on the screen by the " * " symbol.)
9 ÷ 4	(Note that division is displayed by the " / " symbol.)
2 ^ 3	(The ^ key, just above the ÷ key, is for exponents: 2^3.)
-4 + 9	(Use the negation **(-)** key on bottom row for negatives, NOT the subtraction key.)
3 × -6	(Use the negation key, NOT the subtraction key.)
2 + 3 × 4	(Where are the implied parentheses?)
(2 + 3) × 4	(Parenthesis keys are above the **8** and **9** keys.)
2 × 3 + 4	(Again, where are the implied parentheses?)
2 + 3 ÷ 4	(Implied parentheses?)
(2 + 3) ÷ 4	(Parentheses must be used to calculate $\frac{2+3}{4}$.)
2 ÷ 3 × 4	(What would you enter to calculate $\frac{2}{3 \times 4}$?)

When in doubt about which operations are performed first, either try a simple similar example or use parentheses to clarify what you intend. What should you enter to calculate $\frac{4+7}{2+3}$?

$\mathbf{5^2}$	(Press the **5** key first and then the $\mathbf{x^2}$ key on middle left side.)
$\mathbf{(3+4)^2}$	
$\mathbf{5^{-1}}$	(Press **5**, the $\mathbf{2^{nd}}$ key next, and then **EE** key with yellow $\mathbf{x^{-1}}$ label, near the 7 key.)
$\frac{1}{2.4}$ $\frac{1}{2+3}$	(Try each twice: first use the ÷ key, and then use the **2nd** $\mathbf{x^{-1}}$ key.)

√ **4**	(Press yellow **2nd** key first, then $\mathbf{x^2}$ key with yellow √ label.)
$\sqrt{12.25}$	(Parentheses are not necessary.)
$\sqrt{9 + 16}$	(Parentheses are required.)
$\sqrt{9} + \sqrt{16}$	(Is this result the same as the last result?)
π ÷ **2**	(To type π, press yellow **2nd** key first and then ^ key with yellow label "π".)
2 × π	
2 π	(Omit the × key on this and the next three examples;
3 √ **4**	these four examples illustrate "implied" multiplication.)
2 (3 + 4)	
1 / 2 π	(Is the result what you expected?)

Last Entry

Type again the calculation **2 (3 + 4)** and press the **ENTER** key. Now press the yellow **2nd** key and then the **ENTER** key with the yellow **ENTRY** label above it. Notice that your last formula entered reappears on the screen for you to make changes. Use the **arrow** keys to change the "3" to a "5" in order to compute **2 (5 + 4)**. Press the **ENTER** key as usual to carry out the calculation.

Edit your last entry (**2nd ENTRY**) again by first placing the flashing rectangular cursor over the digit "5" (use **left arrow** key). Press the **2nd** key and then the DEL key with the yellow label **INS**. You are now in the "insert" mode instead of the "typeover" mode. Notice that the cursor is a flashing underline rather than a rectangle. Press the **3** key and notice what happens. Then press the **7** key. If you press an **arrow** key, the cursor goes back to the typeover mode as indicated by the flashing rectangular cursor. Press the **ENTER** key. (The cursor does not have to be at the end of the formula.)

Note: Since we've become aware of different cursor styles, watch what happens to the cursor style when you press the yellow **2nd** key. Press the **2nd** key again and watch. If you ever mistakenly press **2nd**, you can cancel by pressing **2nd** again. Look at the cursor to see whether **2nd** is activated. Similarly, watch the cursor style as you slowly press the **ALPHA** key. There is a cycle of three possibilities. Try pressing the **7** key four times after each of the three possibilities. (Don't press the **ENTER** key, and when you have tried all three press the **CLEAR** key.) What did you observe? (After each of the three possibilities, press the **CLEAR** key, but NOT the **ENTER** key.)

Variables

Type the calculation: **2 + 3** **ENTER**. Notice that the answer is 5. Now press the × key and the **4** key. The display screen reads "Ans*4". What is the result when you press **ENTER**?

"Ans" is a variable that stores your last calculated value (as opposed to your last entered formula). Now enter **50** – **Ans**. (Press **2nd** key and then **ANS** key in lower right corner of keyboard.) The value of the variable Ans changes after each new formula calculation.

If you wish to calculate again the last formula entry without editing it, just press the **ENTER** key -- as many times as you wish to perform the calculation.

Example: Press **3** and then the **ENTER** key. The value of Ans is now 3.

Type **2** × **Ans** **ENTER**. (Use the **2nd** and **ANS** keys.) The value of Ans is now 6.

Press the **ENTER** key repeatedly to see the value of Ans repeatedly doubled.

The result of each calculation is automatically stored in the variable Ans. Values can also be stored in single-letter variables by using the **STO▹** key located in the lower left corner of the keyboard. Carry out the following example entries and observe what happens:

5 STO▹ A	**ENTER**	(**STO▹** key flips cursor to **ALPHA**; just press key with label **A**.)
2 + A	**ENTER**	(Press the **ALPHA** key and then the key with label **A**.)
		(The value stored in variable A is used in the calculation.)
A ÷ 2 STO▹ B	**ENTER**	(The value of A/2 is displayed and also stored in variable B.)
A × B	**ENTER**	(Current values of variables A and B are used in calculation.)
7 STO▹ A	**ENTER**	(The value of A is changed, but formula A/2 is not recalculated.)
B	**ENTER**	(Value of B did not change when a new value was stored in A.)

Experiment with other examples using variables to be sure you understand how they work in formulas and calculations.

You should practice repeatedly all the features discussed in this handout (and each later handout) until you are comfortable and proficient with them. You must be able to use these features easily and efficiently. These handouts show you the most important features you will need in calculus. Consult your calculator manual for further details and features.

#2: Functions and Graphing on a TI-86 Calculator

The purpose of handout #2 is to learn how to define, evaluate, and graph functions with your calculator. A TI-86 calculator can work simultaneously with up to 99 user-defined functions. Our first example will be the linear function $y = f(x) = x - 2$. Type or key-press the items in **bold**.

<u>DEFINE</u> the function by pressing the **GRAPH** key and then the **y(x)=** key. Type **x − 2** after the equal sign. Use the key labeled "**x-VAR**" to type the independent variable. (Remember **CLEAR**, **DEL**, **INS**, and **arrow** keys to edit.) Press the **EXIT** key twice or **2nd QUIT** to get back to the Home Screen.

Note 1: In function mode you must use the letter "*x*" as the name of the independent variable for any "*y* =" function, no matter what the independent variable might be named in your actual problem. For example, if your problem uses $g(p) = p^2$, you must use $y1 = x^2$ on the calculator.

<u>EVALUATE</u> the function you have defined by using *y1* as the name of the dependent variable. To type the "*y1*" symbol, press the **2nd alpha** keys to type lower case. Notice the cursor style. Then press the letter **Y** on the zero key. Practice the examples, and observe the output. Remember that our example function is subtraction of 2 from the value of the independent variable *x*.

y1 (3) — (This is standard function notation using "*y1*" instead of "*f* ".)

y1 (6)

5 y1 (4) — (Notice the implied multiplication.)

Note 2: The symbol "*y1*" may be used either as the name of the function (the symbol "*f* " in mathematical function notation) or as the name of the dependent variable (whose current value depends upon the current value of the independent variable "*x*"). The latter is illustrated next.

3 STO▷ x — (We want to evaluate $f(3)$, that is, $y = f(x) = x - 2$ when $x = 3$

y1 — (The value of *y1* depends upon the current value of *x*, namely 3.)

6 STO▷ x : y1 — (A one-line short cut. Use **2nd** and **.** (period) keys to get colon **:** .)

4 STO▷ x : 5 y1 + 3 — (Notice the implied multiplication.)

Note 3: An alternate way to compute the value of a function at a specific point, $f(6)$ for example, is to type **evalF(y1 , x , 3)** on the Home Screen or within an expression. (Press **2nd CALC** to get to **evalF**.) Use **2nd ENTRY** to edit the last expression to: **5 evalF(y1 , x , 4) + 3** and **ENTER**.

<u>GRAPH</u> the function you have defined by pressing the **GRAPH ZOOM MORE ZDECM** keys. The graph of your function is plotted in a viewing window that extends from -6.3 to 6.3 along the *x*-axis and from -3.1 to 3.1 along the *y*-axis. Tick marks are placed every 1 unit along each axis. Press the yellow **2nd** key and then the **WIND**ow key on the top row to see these specifications for the viewing window. The variables *xMin*, *xMax* and *yMin*, *yMax* describe the extent of each coordinate axis. The variables *xScl* and *yScl* describe the numerical distance between tick marks along these axes. The values of the window variables may be changed, but we will not do so at this time.

Press the **GRAPH** key on the top row to see the graph again. Use the **arrow** keys to move the *free-moving cursor* (+ sign) around the viewing window. Notice the *x*- and *y*-coordinates of the point at the center of the cursor. Use the free-moving cursor to write down the coordinates (with comma and parentheses) of three sample points of your choice from the graph of the function.

Press the **TRACE** key on the top row. You will see the X-shaped *trace cursor* on the graph of the function. Notice also the *x*- and *y*-coordinates of the graph point located at the cursor. Use the **right** and **left arrow** keys to move the trace cursor along the graph of the function. Notice for this example function that the *y*-coordinate is always 2 less than the *x*-coordinate. If you press the **ENTER** key, the viewing window will be made to center on the trace cursor. Press the **WIND**ow key to see the new values of the viewing-window variables. Choose **ZOOM MORE ZDECM** to reset the original viewing window. Experiment with what happens when you press the **EXIT**, **QUIT**, **GRAPH**, **TRACE**, **CLEAR**, and **ZOOM ZDECM** keys from various different screens.

Note 4: The trace arrow keys restrict the possible *x*-coordinates of the points that can be specifically computed since the trace cursor moves in jumps from one screen pixel (tiny, square picture element) to another. To compute the value of the function and plot the corresponding graph point at arbitrary values of *x*; within **GRAPH TRACE**, type **2.824**, for example, and press the **ENTER** key. Alternately:

Press **GRAPH MORE MORE EVAL** keys.
Type the desired value of *x* (Eval x = **1.325** for example) and press the **ENTER** key.
(*Alternately*, just press the **TRACE** key, type **1.325**, and press the **ENTER** key.)

Note 5: More than one function can be defined and graphed at the same time. We will use
$y = g(x) = 1 - x$ as our second example function.

Press **GRAPH y(x)= ENTER** to get down to the line "*y2=*". Type **1 - x** and press **2nd GRAPH**.
Press **TRACE**. Move trace cursor to the point where *x*=.7, and watch what happens when you press **up** and **down arrow** keys. Notice the number in upper right corner of screen.
Press **GRAPH TRACE**, type **2.824 ENTER**, and press **up** and **down arrow** keys.
Now press **GRAPH MORE MORE EVAL** and type **2.637 ENTER** for the value of *x*. Again watch what happens when you press the **up** and **down arrow** keys.

#3: Changing the Viewing Window on a TI-86 Calculator

The purpose of handout #3 is to learn how to move the viewing window around the coordinate plane. Our two examples are the following exponential functions: $y = f(x) = 0.5\,(2^x)$ $\quad y = g(x) = 2\,(0.4)^x$

SELECT functions for graphing by pressing the **GRAPH** and **y(x)=** keys, **CLEAR**ing any previously defined functions, and entering **.5 * 2 ^ x** for *y1* and **2 * .4 ^ x** for *y2*. Since the *y1* and *y2* functions have highlighted equal signs, they are "selected" for graphing. Press **GRAPH ZOOM MORE ZDECM** keys and notice the moving dots in the upper right corner of the screen indicating that the calculator is busy working.

Go back to **y(x)=**. Press the **SELCT** key to "deselect" *y1* by turning off the highlighted equal sign. Press the **SELCT** key once again to "select" *y1*. Press the **down arrow** key and deselect *y2*. Press **2nd GRAPH** and observe only one function graph (which one?). Although the deselected *y2* function is not plotted, it is still defined and can be used and evaluated from the Home Screen.

Go back to **y(x)=** and select both *y1* and *y2*. Press **2nd GRAPH**. While graphing, press the **ENTER** key to pause and resume, or the **ON** key to stop. Observe the possible values of *x* and *y* when you use the **arrow** keys to move the free-moving cursor.

CHANGE the viewing **WIND**ow to make *x* vary from -2 (negation key, not subtraction key) to 4 with tick marks every 1 unit and to make *y* vary from -2 to 8 with tick marks every 2 units. **GRAPH**. What is different about the values of *x* and *y* when you now use the **arrow** keys to move the free-moving cursor? The previous **ZOOM ZDECM** had set the window variables so that the pixel steps are 0.1 in all directions. Most other choices for values of the window variables lead to more awkward sizes for the pixel steps. **TRACE** uses the pixel steps for *x*-coordinates and computes the function values to get *y*-coordinates. **EVAL** and **TRACE** also allow you to type in whatever *x*-coordinate you wish, and then they compute the function value to get the *y*-coordinate. Experiment with trying different values of the window variables and using **TRACE** and **EVAL**.

ZOOMING is a shortcut to making certain kinds of changes in the values of the window variables. Choose **ZOOM ZDECM** to set the particularly nice values for the window variables. We can also zoom in and out from whatever is our current viewing window. To set zooming factors press the **ZOOM MORE MORE ZFACT** keys. Change both the *xFact* and *yFact* variables to have the value **2** instead of 4.

<u>ZOOM IN</u>: Press the **ZOOM** and **ZIN** keys, press the **ENTER** key to accept the origin as the zoom center, and when graphing is finished press the **CLEAR** key to stop further zooming. Use the **arrow** keys to move the free-moving cursor, and observe that the pixel steps of 0.05 are exactly half of the previous 0.1 pixel steps. Tick marks appear farther apart on the screen, but actually represent the same numerical distance as before the zoom. Press **GRAPH WIND**ow, and notice that zooming has multiplied the previous min and max values by ½ but has not changed the scale values for tick marks. Change the scale values to .5, and view the **GRAPH**.

<u>ZOOM OUT</u>: Press the **ZOOM** and **ZIN** keys. Use the **arrow** keys to move the move the free-moving cursor to the point (1.5, 1) . Press the **ENTER** key to zoom out centered on this point. Observe how the axes are off-center. When the graphing is finished, press **ENTER** again to zoom out again on the same point. Finally, press **CLEAR** to stop further zooming. In **GRAPH WIND**ow, change both scale variables to 2, and press **GRAPH**.

<u>ZOOM BOX</u>: An alternate method of zooming in is to form a rectangular box to be the new viewing window. Let's apply this method to zoom in on the point of intersection of our two example function graphs. Choose **ZOOM BOX**. Use the **arrow** keys to move the free-moving cursor to any corner of the new viewing rectangle desired. Press the **ENTER** key. Then use the **arrow** keys again to move the cursor to the opposite corner of that rectangle, and press **ENTER**. Keep making new **BOX** selections repeatedly until the x- and y-coordinates of the cursor do not change in, say, the fifth decimal place as the cursor is moved one pixel step in each direction. Write down (using comma and parentheses) the intersection point with coordinates rounded to four decimal places.

Answer: (0.8614, 0.9084)

Note 1: None of the zooming commands affect the scale values, that is the numerical distances between tick marks on the axes. After you have zoomed in and/or out to obtain the viewing window you desire, you may then want to press **WIND**ow and enter suitable values for *Xscl* and *Yscl*. You may also wish to "clean up" the values of *Xmin*, *Xmax*, *Ymin*, and *Ymax*. Press **GRAPH** to view the results.

Note 2: Reset the **ZOOM ZDECM** viewing window. If you wish grid dots displayed within the viewing window, choose **GRAPH MORE FORMT**. Then use **arrow** keys to highlight **GridOn** with the flashing rectangle, and press **ENTER**. Finally, press **GRAPH** to see the result. **GRAPH MORE FORMT** can also be used to turn off the grid dots and make other changes in how the viewing window looks.

#4: Formula Tables and Data Plots on a TI-86 Calculator

The purpose of handout #4 is for you to learn how to use formula tables, data tables, and data plots. Our two example exponential functions have formulas $y = 0.5\,(2^x)$ and $y = 2(0.4)^x$. You should press the **y(x)=** key and enter these formulas for y1 and y2 as before. The example data table is given to the right. We will enter this data into the calculator after we learn how to use a formula table.

x	*y*
-2	3.05
-1	2.55
0	1.94
1	1.64
2	1.25
3	1.07
4	0.84

FORMULA TABLE setup is started by pressing the **TABLE** and **TBLST** (**F2**) keys. Enter **0** for the value of *TblStart* to begin the values of the independent variable *x* in the table and **1** for the value of Δ*Tbl*, to give the step size for *X*. (Leave *Indpnt* as "Auto".) Now press the **TABLE** (**F2**) key to view a table of values for the functions selected and defined by the given formulas. Use the **arrow** keys in a natural way to scroll through the table. Notice that a more precise value of the highlighted entry appears at the bottom of the screen.

Press **TBLST** (**F1**) again. Change the starting value of *x* to -2 and the step size to 0.1. Press **TABLE** (**F1**) to view the table. Use all of the **arrow** keys to explore the new table. **EXIT EXIT** to the Home Screen.

DATA TABLE entry is started by pressing the yellow **2nd** and **STAT** keys and choosing **Edit** (**F2**). The first two columns in the table give *lists* of corresponding data values for variables *x* and *y*. Enter the "*x*" data values from the example data table above into the "*xStat*" column on the calculator. Press the **ENTER** or **down arrow** key to go to the next value. Press the **right arrow** key to go to the "*yStat*" column, and enter the "*y*" data values from the example data table above. Use **arrow** keys to move to any entry you wish to correct. While you are editing an individual entry, the **CLEAR**, **DEL**, and **INS** keys work as they have in the past. If an entry is highlighted and you have not started to edit the entry, the **DEL** key removes that entry from the list and pushes others below it up. Pressing the **INS** key will insert a zero (which can be edited) and push other entries down the list. Experiment with the editing features until you are comfortable with how they work. It is important to remember that all of the numbers in a data table are there by individual entry, not from a formula calculation.

PLOT THE DATA from the data table as follows:

1. Deselect all of the functions defined by formulas under **GRAPH y(x)=**. You may do this either separately for each function (by using **SELCT** to remove the highlight on the equal signs as in an earlier handout) or all at once (by using **MORE ALL --**).

2. Press the **2nd STAT** and **PLOT** keys (yellow label on the + key), and choose **PLOT1**. Highlight **On** and press the **ENTER** key to select **On**. If not already selected, the following options should be chosen by using the **down arrow** and pressing **ENTER**.:

   Type: **SCAT**ter
   Xlist: **xStat**
   Ylist: **yStat**
   Mark: □

3. Press **GRAPH ZOOM MORE** and **ZDATA** keys to automatically set a viewing window that includes all of the data points. Press **WIND**ow, enter suitable scale values, and modify the other window variables as desired. For example, change variables *yMin* to **0** and *ymMax* to **3.5**. Press the **F5** (**GRAPH**) and **CLEAR** keys to view the data in your new window. Notice that the graph of the data looks somewhat like a decreasing exponential function.

MODEL THE DATA using a function defined by a formula. You can use trial and error to determine possible values for the parameters a and b in the family of exponential functions with formula

$y = b \cdot a^x$ so that the graph of the function fits the data points as best as you can make it. The specific formula should be entered as one of the selected **y(x)=** functions. For example, start by entering the formula **2.1 × 0.7 ^ x** for the function y3. View the (**2nd**) **GRAPH** to see how well this specific model fits the given data. Modify the values of the parameters a and b to try to get a better fit.

TURN OFF THE DATA PLOT by pressing the (**2nd**) **STAT**, **PLOT** (**F3**), **PlOff** (**F5**), and **ENTER** keys. (If you do not see the word "Done" on the screen, press **ENTER** again.)

IMPORTANT: BE SURE YOU TURN OFF THE DATA PLOT WHEN YOU FINISH THIS SECTION.

Note: A family of functions can be graphed using a *list* for the parameter. For example, graph the family $f(x) = x^p$ by entering y4 = **x ^ { .5, 1, 1.5, 2 }** ({ } are **F1 F2** keys under **2nd LIST** menu.)

#5: Left and Right Sums on a TI-86 Calculator

The purpose of handout #5 is to implement the computation of left- and right-hand sums on the calculator. Suppose we wish to estimate the area under the graph of $f(x)$ over the interval from $x=a$ to $x=b$ by forming the appropriate left-hand and right-hand sums for various values of n. For any choice of n, the increment in x would be $h = \frac{b-a}{n}$, and the equally spaced values of x would be

$$\begin{cases} a = x_0 \\ \quad x_1 = x_0 + h = a + h \\ \quad x_2 = x_1 + h = a + 2h \\ \quad x_3 = x_2 + h = a + 3h \\ \quad \vdots \qquad \vdots \qquad \vdots \\ b = x_n = x_{n-1} + h = a + nh \end{cases}$$

Given $f(x)$, a, b, and a choice for n, the left- and right-hand sums, L and R, may be written:

$$L = \sum_{i=0}^{n-1} f(a+ih) \cdot h \quad \text{and} \quad R = \sum_{i=1}^{n} f(a+ih) \cdot h, \quad \text{where } h = \frac{b-a}{n}.$$

Notice that the only dissimilarity between the left-hand sum and the right-hand sum is in the lower and upper limits of summation. Finally, as the value of n becomes arbitrarily large, the values of L and R both approach the area under the graph of the function $f(x)$ over the interval $a \le x \le b$.

To implement the computation of left-hand sums on the calculator, we will write a program, that is, a list of the steps for the calculator to do. We need to type the program into the calculator only once. Then to compute a left-hand sum, we define the particular function and request the calculator to perform the program steps ("execute the program"). We can similarly implement the computation of right-hand sums. A brief *outline* of the program we will create later is as follows:

a. Prompt for values of the parameters a, b, and n.

b. Compute the increment, $h = \frac{b-a}{n}$.

c. Starting with a sum of zero, repeatedly add each new product $f(a+ih) \cdot h$ to the previously accumulated sum.

d. Display the final sum.

<u>CREATE NEW PROGRAMS</u> by typing program instructions into the calculator:

1. Press the **PRGM** key and select **EDIT**.
2. Type in the name of your new program (**LHS**), and press the **ENTER** key. (Note that the flashing cursor is set into the **ALPHA** mode so you can just press the keys corresponding to the letters in the program name.)

3. Type in the program itself using the key-press hints given in parentheses. Press the **ENTER** key at the end of each line. Remember to press the **ALPHA** key to type an alphabetic letter. If you need to make corrections, use the **INS**, **DEL**, and **arrow** keys in the natural way.

   Prompt A , B , N (Press the **I/O** menu key and then the **Promp** menu key.)

   (B - A) / N → H

   0 → L

   For(I , 0 , N - 1) (Press the **CTL** menu key and then the **For** menu key.)

   L + y1(A + I * H) * H → L (Press the **2nd** **ALPHA** and **y** keys to get lower case.)

   End (Press the **CTL** menu key and select **End**.)

   Disp L (Press the **I/O** menu key and select **Disp**.)

   Stop (Press the **CTL** menu key, press **MORE** twice, and select **Stop**.)

4. When you have finished typing in the program, press the **2nd** **QUIT** key.
5. To type in the program for computing right-hand sums, repeat the instructions in steps 1-4. Use **RHS** for the name, replace "**L**" by "**R**" everywhere, and change the For statement to: **For(I , 1 , N)**
6. If you need to modify a program after you have typed it into the calculator, press the **PRGM** and **EDIT** keys, and select the program you want to modify. Use the **INS**, **DEL**, and **arrow** keys in the natural way to help you make any changes. When you are finished, press **2nd** **QUIT** key.

<u>EXECUTE PROGRAMS</u> LHS and RHS to approximate, for example, the area under the graph of $f(x) = x^3$ over the interval from $x = 1$ to $x = 3$, using $n = 100$ subdivisions:

1. Enter the formula x^3 as the *y1* function. Remember that LHS and RHS were written to use only the *y1* function.
2. Press the **PRGM** and **NAMES** keys. Select the **LHS** program, and press the **ENTER** key to execute the program from the Home Screen.
3. Enter the values **1**, **3**, and **100** (one value at each question mark) for the values of the parameters *a*, *b*, and *n*.
4. <u>Wait</u> until the calculation is **Done**. (Answer should be 19.7408.)
5. Repeat steps 2-4 to compute the corresponding RHS. (Answer should be 20.2608.)
6. Enter the formula **(L + R) / 2** on the Home Screen to compute the average of the left- and right-hand sums. (Answer should be 20.0008.)

#6: Calculus Features on a TI-86 Calculator

In addition to the operations that we have previously learned (see earlier handouts), there are some powerful features on a TI-86 calculator that provide shortcuts to various calculus calculations. There are two major ways to carry out most of these calculations:

a. Commands executed from the home screen or included in **y(x)=** function definitions, and
b. Interactive calculations on the graphing screen.

Consult your TI-86 calculator manual for further details of features discussed below and many more!

For this handout, define example functions $f(x) = x^3 - 3x^2 + 2.5$ $g(x) = x - 0.4$ and respectively. Graph both functions using **ZOOM MORE ZDECM**. Use ▹ and ◃ to change the value of x. Type or key-press items in **bold**.

1. *Evaluate a function*. For example, compute $f(1.6183)$ and $g(1.6183)$. (Ans: -1.11853 and 1.2183)

   Home Screen: Enter **y1(1.6183)** and **y2(1.6183)** or **CALC evalF(y2 , x , 1.6183)** as before.

   Interactive: **GRAPH MORE MORE EVAL** Eval x = **1.6183** Use △ and ▽ to switch functions.

   Notes: **GRAPH TRACE** gives function evaluation at pixel-based and entered values of x.
   TABLE provides another way to evaluate functions.

2. *Find zeros of a function*. For example, the first of three zeros of $f(x)$ lies between -1 and 0 at about -0.5 for x.

   Interactive 1: Press the **2nd SOLVER** key, and type eqn: **y1 = 0 ENTER**. (**ALPHA STO▹** for =)
   x = **-.5 ENTER** bound = **{ -1 , 0 }** (Use **2nd LIST** to type { and }.)
   Arrow up to the line "x = -.5" in order to solve for the variable x.
   Press **SOLVE**, and see x = -0.81003792923396 is a zero of $f(x)$.
   Change the bound values and arrow to "x =" to find the other two zeros.

   Interactive 2: **GRAPH MORE MATH ROOT** Use △ and ▽ to select function *y1* (see right corner).
   LeftBound? x= **-1** RightBound? x= **0** Guess? x= **-.5** (other values possible)
   See value x = -0.810379292 as a zero of $f(x)$.

3. *Find intersection points of two graphs*. For example, the first of the three intersection points of $f(x)$ and $g(x)$ is at about -1 for x.

   Interactive 1: **2nd SOLVER**. Press **up arrow** key to edit eqn: **y1 – y2 = 0 ENTER**.
   Use initial guess x = **-1** and bound = **{ -2 , 0 }**, go up to "x =", and **EXIT SOLVE**.
   See x = -0.98738081838218 at the intersection point. **QUIT** to the Home Screen.
   y1 ENTER. See y = -1.38738081838 at the intersection point.

   Interactive 2: **GRAPH MORE MATH MORE ISECT** Use △, ▽ and **ENTER** to select two curves.
   LeftBound? x= **-2** RightBound? x= **0** Guess? x= **-.1** (other values possible)

   Note: Repeated use of **TRACE** and **ZOOM** (as we did in earlier handout) is a much longer process for finding zeros of a function and intersection points.

4. *Find local maxima and minima of a function on an interval.* For example, find the single local maximum or single local minimum of the function $f(x)$ on the interval [1,3], if there is one.

Home Screen: **CALC MORE fMax(y1,x,1,3)** See 3 (rounded) as value of *x* making $f(x)$ largest.

CALC evalF(y1, x, Ans) See 2.5 (rounded) as that maximum value of $f(x)$.

Interactive: **GRAPH MORE MATH FMIN** Use △, ▽ to select *y1*, and press **ENTER**.

LeftBound? x= **1** RightBound? x= **3** Guess? x= **2.5** (other values possible)

See x = 2 (rounded) and y = -1.5 at minimum point for $f(x)$ over interval [1,3].

5. *Find and graph the derivative of a function.* For example, compute $f'(2.5)$ and graph $f'(x)$.

Home Screen: **2nd CALC nDer(y1, x, 2.5)** See 3.750001 as approximate value of $f'(2.5)$.

Note: nDer uses a central difference quotient with 0.001 as the default value of *h*.

2nd MEM TOL δ= **.0005 EXIT ENTER** gives 3.75000025 with 0.0005 as *h* value.

y(x)= Screen: Define y3= **2nd CALC nDer(y1, x, x)** and **GRAPH ZOOM MORE ZDECM** to graph.

y3 can be used as $f'(x)$ for computation and graphing. Deselect *y3* in **y(x)=**.

Interactive: **GRAPH MORE MATH dy/dx**. Use △ and ▽ to select function *y1* (see upper right).

Set x= **2.5 ENTER**. See 3.75 as approximate value of dy/dx at given point.

6. *Find the definite integral of a function over an interval.* For example, compute $\int_{0.2}^{2.0} f(x)\,dx$.

Home Screen: **2nd CALC fnInt(y1, x, .2, 2)** and see 0.5076 as value of the definite integral.

Interactive: **GRAPH MORE MATH** $\int f(x)$ Use △ and ▽ to select function *y1* (upper right).

LowerLimit? x= **.2** UpperLimit? x= **2** See 0.5076 as value of integral.

7. *Draw on the graphing screen.*

Use **GRAPH MORE DRAW MORE MORE CLDRW** to clear previous drawings. Deselect *y*2 in **y(x)=**.

GRAPH MORE DRAW MORE DrawF y1(x)−2 ENTER. Temporary graph of $f(x)-2$. **CLDRW**

GRAPH MORE DRAW MORE MORE MORE DrInv y1 ENTER. Graph of inverse relation for $f(x)$.

This example inverse relation is not a function since $f(x)$ is not invertible. **CLDRW**

8. *Draw tangent lines to graph of a function.* For example, draw the tangent line to the graph of $f(x)$ at $x = 2.2$. Be sure that only $f(x)$ is selected on the **Y=** screen.

Home Screen: **GRAPH MORE DRAW MORE MORE MORE TanLn(y1, 2.2) ENTER**. (**CLDRW**)

Graphing Screen: **GRAPH MORE MATH MORE MORE TANLN**. Set x= **2.2**, and press **ENTER**.

#7: Differential Equations on a TI-86 Calculator

The purpose of handout #7 is to implement graphing the slope field of a differential equation and sketching an approximate solution curve using Euler's method. The programs for graphing a slope field and for using Euler's method may either be typed into your calculator by hand or transferred from another calculator by linking. Type or key-press items in **bold**. (See calculator manual for more differential equations features.)

TRANSFER THE PROGRAMS BY LINKING

1. Attach the LINK cable to both calculators. Be sure the cable is pushed in *very firmly*.
2. Turn on both calculators, and press **2nd LINK** on each. On the "destination" calculator, press the **RECV** key to be ready and *waiting...* to **RECEIVE** the programs.
3. On the "source" calculator, select **SEND PRGM**, highlight the desired program name(s), and press **SELCT** to select each one desired. Those items selected for transmission are marked with a ■. Press the **XMIT** key to transmit the selected program(s).

EXECUTE THE PROGRAMS

0. First be sure your calculator has the program, either transferred by linking (described above) or typed in (described below).
1. In the window $-4 \le x \le 4$ and $-3 \le x \le 3$, let's use the example differential equation $\frac{dy}{dx} = x + y$
2. Press **GRAPH y(x)=** and enter the expression **x + y** after the "y1=". Press **WIND** and enter the appropriate values for the *x* and *y* intervals, using a scale value of 1 in each direction.
3. Press the **PRGM**, **NAMES**, and **SLOPE** keys. Press the **ENTER** key to execute the **SLOPEFLD** program from the Home Screen.
4. Watch the slope field being graphed. When finished, press **2nd QUIT**.
5. Press the **PRGM**, **NAMES**, and **EULER** keys. Press the **ENTER** key to execute the **EULER** program from the Home Screen.
6. Type **1** and **ENTER** at the first prompt to keep the plotted slope field in the viewing window. Enter **-1** for the initial value of *x* and **.2** for the initial value of *y*. Enter **2** to graph a solution curve in both directions from the initial point. When finished, press **2nd QUIT**.
7. Press the **ENTER** key to repeat the Euler program, type **0** at the first prompt to clear the viewing window, and use the same initial point to make a graph in both directions. **2nd QUIT**.
8. Without clearing the viewing window, repeat the Euler program several times using different initial points and choices for the direction(s) to graph from the initial point.

TYPE IN THE PROGRAMS

1. The slope field program is adapted from a program by Mark Howell that appears on page 113 of *Technology Resource Manual for Calculus* by Finney, Thomas, Demana, and Waits.

2. See handout #5 for general instructions on creating new programs on the TI-86. Note that the *x* and *y* variables are lower case.

3. Some of the new key-press combinations for the TI-86 are: For the lower case **x**, **y** variables and for **FnOff**, use **GRAPH VARS** and **MORE MORE**. For **ClDrw** and **Line**, use **GRAPH MORE DRAW** and **MORE**. For **DispG** and **Input**, use **I/O**. For **Lbl**, **Goto**, **For**, **If**, **While**, use **CTL** and **MORE**. For <, >, and ==, use **2nd TEST**. For **xMin**, **xMax**, **yMin**, **yMax**, and **Δx**, use **CATLG-VARS MORE MORE WIND** and **PAGE↓**. For **"**, use **2nd STRNG**. For **abs**, use **2nd MATH NUM**.

SLOPE FIELD PROGRAM (**SLOPEFLD**)

```
14 → L
18 → W
( yMax − yMin ) / L → V
( xMax − xMin ) / W → H
FnOff
ClDrw
DispG
yMin + V / 2 → y
For ( R , 1 , L )
xMin + H / 2 → x
For ( C , 1 , W )
y1 → M
- M * H / 2 + y → S
M * H / 2 + y → T
If abs ( T − S ) > V
Then
y + V / 2 → T
y - V / 2 → S
( T - y ) / M + x → Q
( S - y ) / M + x → P
Else
x - H / 2 → P
x + H / 2 → Q
End
y → Y
Line ( P , S , Q , T )
Y → y
x + H → x
End
y + V → y
End
```

EULER'S METHOD PROGRAM (**EULERS**)

```
Disp "TO CLEAR WINDOW"
Input "ENTER 0 : ", C
If C == 0
ClDrw
FnOff
Input "INITIAL x : ", P
Input "INITIAL y : ", S
Disp "1 LEFT , 3 RIGHT , "
Input "2 BOTH : ", C
If C < 2
Goto L2
P → x
S → y
While x < xMax
x + Δx → Q
y + y1 * Δx → T
Line ( x , y , Q , T )
Q → x
T → y
End
Lbl L2
If C > 2
Goto L4
P → x
S → y
While x > xMin
x − Δx → Q
y − y1 * Δx → T
Line ( x , y , Q , T )
Q → x
T → y
End
Lbl L4
DispG
Stop
```